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HOMOGENIZATION AND LINEAR
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by
Gilles A. Francfort

SUDAM Report No. 81-4

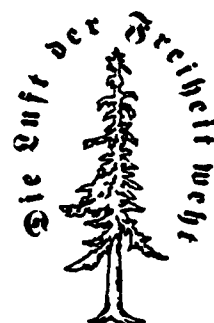
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ABSTRACT

We study homogenization of linear dynamic thermoelasticity with rapidly varying coefficients, using a semi-group approach. The resulting homogenized problem exhibits an unusual change in initial temperature.

A formal asymptotic analysis predicts fast time oscillations in the temperature field. These oscillations explain the temperature shift, and show that, for our problem, weak convergence in time is the best convergence that one can obtain.

INTRODUCTION

We discuss the problem of "homogenizing" the equations of linear thermoelasticity when the mechanical and thermal properties are periodic and rapidly varying. Following Bensoussan - Lions - Papanicolaou [1] and Sanchez-Palencia [7] and using a semi-group approach, we show rigorously that, as the period of the coefficients goes to zero, the solution of these equations converges to the solution of a related constant coefficient problem, the homogenized problem. Then using a formal multiple-scales method, we give what we believe to be a satisfying interpretation of some surprising features of the results.

Thermoelastic behaviour is characterized by the coupling of hyperbolic equations of motion and a parabolic heat equation. This leads to several interesting phenomena in the homogenization process. Fast oscillations in the temperature field are observed; their amplitude remains finite as the period goes to zero. Thus the solutions can only converge in a weak sense in time to a slowly varying homogenized solution.

Furthermore, the initial data for the homogenized problem are related to the initial data of the inhomogeneous problem by a linear transformation that is not a projection. We know of no other examples of such phenomenon.

In section 1, we formulate and prove the existence of a homogenized thermoelastic medium. Section 2 contains the more formal arguments and the fast oscillations results, which are at the root of the observed change in initial data.

1. HOMOGENIZATION OF THE THERMOELASTIC PROBLEM

To cut down on the overwhelmingly cumbersome notations that characterize thermoelasticity, we will place ourselves in a scalar setting, that is one where the displacement field is taken to be scalar-valued. Duvaut-Lions [2] show, using Korn's theorem, that this is no loss of generality.

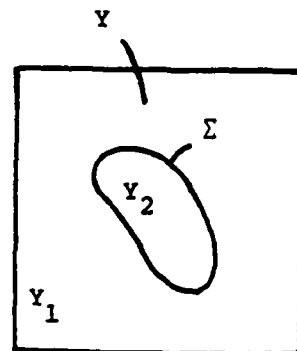
We consider a domain Ω of \mathbb{R}^n . The degree of smoothness of $\partial\Omega$ will depend on the type of boundary conditions adopted. We will always assume that $\partial\Omega$ is smooth enough for one to be in position to apply Rellich's theorem on compact imbeddings of Sobolev spaces (Folland [3], Chapter 6).

We will refer to $Y = \prod_{i=1}^n]0, y_i^o[$ as to the "reference cell"; $|Y|$ is its volume.

If Σ is a smooth hypersurface dividing Y into Y_1 and Y_2 , we define $a_{ij}(y)$, $\lambda_{ij}(y)$, $\alpha_i(y)$, $\beta(y)$, $\rho(y)$ to be real Y -periodic functions, smooth and bounded on the closure of Y_1 and Y_2 but with Σ as potential surface of discontinuity.

Furthermore, $a_{ij}(y)$, $\lambda_{ij}(y)$ are assumed to be symmetric, strongly elliptic on Y , that is that there exists $\alpha > 0$ such that:

$$(1.1) \quad a_{ij}(y) \text{ (resp. } \lambda_{ij}(y)) \xi_i \xi_j \geq \alpha \xi_i^2 \text{ on } Y$$



$\beta(y)$ and $\rho(y)$ are bounded away from zero. We finally choose α such that α^{-1} is a common upper bound to the L_∞ -norms of the coefficients. We extend all coefficients to all of \mathbb{R}^n by periodicity. Our equations are (Kupradze [5]):

$$(1.2) \quad \begin{cases} \rho\left(\frac{x}{\varepsilon}\right) \frac{\partial^2 u^\varepsilon}{\partial t^2} = \frac{\partial}{\partial x_i} \left(a_{ij}\left(\frac{x}{\varepsilon}\right) \left(\frac{\partial u^\varepsilon}{\partial x_j} - \alpha_j\left(\frac{x}{\varepsilon}\right) \tau^\varepsilon \right) \right) \\ \beta\left(\frac{x}{\varepsilon}\right) \frac{\partial \tau^\varepsilon}{\partial t} = \frac{\partial}{\partial x_i} \left(\lambda_{ij}\left(\frac{x}{\varepsilon}\right) \frac{\partial \tau^\varepsilon}{\partial x_j} \right) - a_{ij}\left(\frac{x}{\varepsilon}\right) \alpha_j\left(\frac{x}{\varepsilon}\right) \frac{\partial^2 u^\varepsilon}{\partial t \partial x_i} \end{cases}$$

For the sake of simplicity we will only consider Dirichlet boundary conditions throughout.

$$(1.3) \quad u^\varepsilon = 0, \quad \tau^\varepsilon = 0 \quad \text{on } \partial\Omega$$

And for initial conditions, we will have:

$$(1.4) \quad u^\varepsilon(x, 0) = f(x), \quad \frac{\partial u^\varepsilon}{\partial t}(x, 0) = g(x), \quad \tau^\varepsilon(x, 0) = k(x)$$

Our goal is to study the behavior of u^ε and τ^ε as ε , the period, goes to zero.

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We define H to be:

$$(1.5) \quad H = H_0^1(\Omega) \times L_2(\Omega) \times L_2(\Omega)$$

On H , we define the operator A_ε :

$$(1.6) \quad A_\varepsilon = \begin{pmatrix} 0 & 1 & 0 \\ \frac{1}{\rho(\frac{x}{\varepsilon})} \frac{\partial}{\partial x_i} (a_{ij}(\frac{x}{\varepsilon}) \frac{\partial}{\partial x_j}) & 0 & -\frac{1}{\rho(\frac{x}{\varepsilon})} \frac{\partial}{\partial x_i} (a_{ij}(\frac{x}{\varepsilon}) \alpha_j(\frac{x}{\varepsilon})) \\ 0 & -\frac{1}{\beta(\frac{x}{\varepsilon})} a_{ij}(\frac{x}{\varepsilon}) \alpha_j(\frac{x}{\varepsilon}) \frac{\partial}{\partial x_i} & \frac{1}{\beta(\frac{x}{\varepsilon})} \frac{\partial}{\partial x_i} (\lambda_{ij}(\frac{x}{\varepsilon}) \frac{\partial}{\partial x_j}) \end{pmatrix}$$

with domain

$$(1.7) \quad D(A_\varepsilon) = \{U = (u, u_\varepsilon, \tau) \in H_0^1(\Omega) \times L_2(\Omega) \times H_0^1(\Omega)\}$$

such that $A_\varepsilon U \in H$ in a weak sense }

Then the following proposition holds:

Proposition 1.1

A_ε generates in H a strongly continuous semi-group of operators $S_\varepsilon(t)$ such that :

$$(1.8) \quad || S_\varepsilon(t) || \leq \alpha^{-1} \quad (\forall t > 0)$$

Proof

We first consider for a fixed ε the norm

$$(1.9) \quad |U|_{\varepsilon}^2 = \int_{\Omega} [a_{ij}(\frac{x}{\varepsilon}) \frac{\partial u}{\partial x_j} \frac{\partial \tilde{u}}{\partial x_i} + \rho(\frac{x}{\varepsilon}) u_t \tilde{u}_t + \beta(\frac{x}{\varepsilon}) \tau \tilde{\tau}] dx$$

where \sim denotes complex conjugate.

In view of the properties of the coefficients, $|\cdot|_{\varepsilon}$ is indeed a norm on H , equivalent to the natural Sobolev norm on H , noted $\|\cdot\|$, that is, if U is in H :

$$(1.10) \quad \alpha \|U\|^2 \leq |U|_{\varepsilon}^2 \leq \alpha^{-1} \|U\|^2$$

In the norm $|\cdot|_{\varepsilon}$, A_{ε} generates a semi-group of contractions. Indeed, the domain $D(A_{\varepsilon})$ is dense, since, though $C_0^{\infty}(\Omega)$ functions do not belong to it, $C_0^{\infty}(\Omega)$ functions whose normal derivatives are 0 on the only possible surfaces of discontinuity for the coefficients (i.e. the ε -scaled versions of Σ in each of the cells making up Ω) do belong to the domain $D(A_{\varepsilon})$. Checking that A_{ε} is closed, that the range of $(1-A_{\varepsilon})$ is H itself and that A_{ε} is dissipative offers no special difficulties (see Francfort [4] for full details). Note that the measure of the dissipation,

$$(1.11) \quad \operatorname{Re}(A_{\varepsilon}U, U) = - \operatorname{Re} \left(\int_{\Omega} \lambda_{ij}(\frac{x}{\varepsilon}) \frac{\partial \tau}{\partial x_j} \frac{\partial \tilde{\tau}}{\partial x_i} dx \right) \leq - \alpha |\nabla \tau|_{L_2(\Omega)}^2$$

(in view of the properties of the λ_{ij} 's), is precisely the physical dissipation due to heat fluxes through the boundary.

The result then follows from the application of Lumer-Phillips's theorem (Yosida [8], Chapter 9). Therefore,

$$(1.12) \quad |S_\varepsilon(t)U|_\varepsilon \leq |U|_\varepsilon \quad \text{for any } U \text{ in } H$$

and thus, using (1.10),

$$(1.13) \quad \|S_\varepsilon(t)U\| \leq \alpha^{-1} \|U\|$$

which completes the proof.

We now leave the time dependent formulation and examine the behavior of the resolvent of A_ε , $R_\lambda(A_\varepsilon)$ as ε goes to 0. At the end of this section we will reintroduce the time dependence by using some basic properties of semi-groups.

It is a direct consequence of (1.8) (Yosida [8], Chapter 9) that the right half complex plane belongs to the resolvent set of A_ε , for every ε . Let us consider $F = (f, g, k)$ to be an element of H . Taking λ to be real strictly positive, we have the following string of equivalences:

$$(1.14) \quad R_\lambda(A_\varepsilon)F = U_\varepsilon, \quad (U_\varepsilon = (u^\varepsilon, u_t^\varepsilon, \tau^\varepsilon)) \quad \longleftrightarrow$$

$$\begin{cases} \lambda u^\varepsilon - u_t^\varepsilon = f \\ \rho(\frac{x}{\varepsilon}) \lambda u_t^\varepsilon - \frac{\partial}{\partial x_i} (a_{ij}(\frac{x}{\varepsilon}) (\frac{\partial u^\varepsilon}{\partial x_j} - \alpha_j(\frac{x}{\varepsilon}) \tau^\varepsilon)) = \rho(\frac{x}{\varepsilon}) g \\ \beta(\frac{x}{\varepsilon}) \lambda \tau^\varepsilon - \frac{\partial}{\partial x_i} (\lambda_{ij}(\frac{x}{\varepsilon}) \frac{\partial \tau^\varepsilon}{\partial x_j}) + a_{ij}(\frac{x}{\varepsilon}) \alpha_j(\frac{x}{\varepsilon}) \frac{\partial u_t^\varepsilon}{\partial x_i} = \beta(\frac{x}{\varepsilon}) k \end{cases} \longleftrightarrow$$

$$(1.15) \quad \begin{cases} \lambda u^\varepsilon - u_t^\varepsilon = f \\ \lambda^2 \rho(\frac{x}{\varepsilon}) u^\varepsilon - \frac{\partial}{\partial x_i} (a_{ij}(\frac{x}{\varepsilon}) (\frac{\partial u^\varepsilon}{\partial x_j} - \alpha_j(\frac{x}{\varepsilon}) \tau^\varepsilon)) = \rho(\frac{x}{\varepsilon}) (\lambda f + g) \\ \lambda \beta(\frac{x}{\varepsilon}) \tau^\varepsilon - \frac{\partial}{\partial x_i} (\lambda_{ij}(\frac{x}{\varepsilon}) \frac{\partial \tau^\varepsilon}{\partial x_j}) + \lambda a_{ij}(\frac{x}{\varepsilon}) \alpha_j(\frac{x}{\varepsilon}) \frac{\partial u^\varepsilon}{\partial x_i} = \beta(\frac{x}{\varepsilon}) k + a_{ij}(\frac{x}{\varepsilon}) \alpha_j(\frac{x}{\varepsilon}) \frac{\partial f}{\partial x_i} \end{cases}$$

The last two equations (1.15) have a unique solution $v^\varepsilon = \lambda u^\varepsilon, \tau^\varepsilon$ in $(H_0^1(\Omega))^2$, since the Dirichlet form d_ε defined as:

$$(1.16) \quad d_\varepsilon((v^\varepsilon, \tau^\varepsilon), (\zeta, \eta)) = \frac{1}{\lambda} \int_\Omega a_{ij}(\frac{x}{\varepsilon}) \frac{\partial v^\varepsilon}{\partial x_i} \frac{\partial \zeta}{\partial x_j} dx + \\ \lambda \int_\Omega \rho(\frac{x}{\varepsilon}) v^\varepsilon \zeta dx - \int_\Omega a_{ij}(\frac{x}{\varepsilon}) \alpha_j(\frac{x}{\varepsilon}) \tau^\varepsilon \frac{\partial \zeta}{\partial x_i} dx + \int_\Omega \lambda_{ij}(\frac{x}{\varepsilon}) \frac{\partial \tau^\varepsilon}{\partial x_j} \frac{\partial \eta}{\partial x_i} dx \\ + \lambda \int_\Omega \beta(\frac{x}{\varepsilon}) \tau^\varepsilon \eta dx + \int_\Omega a_{ij}(\frac{x}{\varepsilon}) \alpha_j(\frac{x}{\varepsilon}) \frac{\partial v^\varepsilon}{\partial x_i} \eta dx$$

is strictly coercive on $(H_0^1(\Omega))^2$, in view of the properties of the coefficients.

If we manage to find a limit for $u^\varepsilon, \tau^\varepsilon$ as ε goes to zero, then going back up through the string (1.14) will enable us to get the limit of $R_\lambda(A_\varepsilon)F$.

Performing the limiting process in (1.14) is the task of the homogenization method. Rather than going through all the lengthy details of the argument, we merely mention the different steps that were performed, underlining only the ones that are not totally standard. For further details the reader is to refer to Bensoussan-Lions-Papanicolaou [1], Chapter 1, especially Sections 3, 9, and 13, or, for our problem to Francfort [4].

First one shows that u_ε and τ_ε are bounded in $(H_0^1(\Omega))^2$, which immediately implies the existence of a weakly convergent subsequence in $(H_0^1(\Omega))^2$ converging to (u, τ) . Since we ultimately show that any convergent subsequence converges to the same limit we do not distinguish between the sequence and subsequences of this sequence.

Then, defining

$$(1.17) \quad \begin{cases} \sigma_i^\varepsilon = a_{ij}(\frac{x}{\varepsilon}) \left(\frac{\partial u^\varepsilon}{\partial x_j} - \alpha_j(\frac{x}{\varepsilon}) \tau^\varepsilon \right) & \text{the stress,} \\ \kappa_i^\varepsilon = \lambda_{ij}(\frac{x}{\varepsilon}) \frac{\partial \tau^\varepsilon}{\partial x_j} & \text{the heat flux,} \\ v^\varepsilon = a_{ij}(\frac{x}{\varepsilon}) \alpha_j(\frac{x}{\varepsilon}) \frac{\partial u^\varepsilon}{\partial x_j} \end{cases}$$

it is easy to conclude that these quantities converge weakly in $L_2(\Omega)$ to σ_i , κ_i , v , which in turn satisfy:

$$(1.18) \quad \begin{cases} \bar{\rho} \lambda^2 u - \frac{\partial \sigma_i}{\partial x_i} = \bar{\rho}(\lambda f + g) \\ \bar{\rho} \lambda \tau - \frac{\partial \kappa_i}{\partial x_i} + \lambda v = \bar{\rho} k + \overline{a_{ij} \alpha_j} \frac{\partial f}{\partial x_i} \end{cases},$$

where, from now on, $\overline{\quad}$ will denote the Y -average $\frac{1}{|Y|} \int_Y dy$.

It remains to determine σ_i , κ_i , and v . This is the core of homogenization. To this effect we define $\chi_k(y)$, $\Theta_k(y)$, $\Psi(y)$ to be the unique periodic solutions, up to a constant, in $H^1(Y)$ of:

$$(1.19) \quad \begin{cases} -\frac{\partial}{\partial y_i} (a_{ij}(y) \frac{\partial \chi_k}{\partial y_j}) = -\frac{\partial a_{ik}}{\partial y_i}(y) \\ -\frac{\partial}{\partial y_i} (\lambda_{ij}(y) \frac{\partial \theta_k}{\partial y_j}) = -\frac{\partial \lambda_{ik}}{\partial y_i}(y) \\ -\frac{\partial}{\partial y_i} (a_{ij}(y) \frac{\partial \Psi}{\partial y_j}) = -\frac{\partial}{\partial y_i} (a_{ij}(y) \alpha_j(y)) \end{cases}$$

Ψ can be considered as non standard with respect to the "classical" case. The functions:

$$(1.20) \quad w_k^\varepsilon = x_k - \varepsilon \chi_k\left(\frac{x}{\varepsilon}\right), \quad z_k^\varepsilon = x_k - \varepsilon \theta_k\left(\frac{x}{\varepsilon}\right),$$

satisfy:

$$(1.21) \quad \begin{cases} \int_{\Omega} a_{ij}\left(\frac{x}{\varepsilon}\right) \frac{\partial w_k^\varepsilon}{\partial x_j} \frac{\partial \tilde{w}}{\partial x_i} dx = 0 \\ \int_{\Omega} \lambda_{ij}\left(\frac{x}{\varepsilon}\right) \frac{\partial z_k^\varepsilon}{\partial x_j} \frac{\partial \tilde{\mu}}{\partial x_i} dx = 0, \text{ for any } w, \mu \text{ in } H_0^1(\Omega). \end{cases}$$

Taking w and μ to be $C_0^\infty(\Omega)$ functions and making use of (1.16), (1.21), we have:

$$(1.22) \quad \begin{aligned} d_\varepsilon((\lambda u^\varepsilon, \tau^\varepsilon), (w w_k^\varepsilon, \mu z_k^\varepsilon)) - \int_{\Omega} a_{ij}\left(\frac{x}{\varepsilon}\right) \frac{\partial w_k^\varepsilon}{\partial x_i} \frac{\partial (\tilde{w} u^\varepsilon)}{\partial x_j} dx \\ - \int_{\Omega} \lambda_{ij}\left(\frac{x}{\varepsilon}\right) \frac{\partial z_k^\varepsilon}{\partial x_j} \frac{\partial (\tilde{\mu} \tau^\varepsilon)}{\partial x_i} dx = \int_{\Omega} \rho\left(\frac{x}{\varepsilon}\right) (\lambda f + g) \tilde{w} w_k^\varepsilon dx \\ + \int_{\Omega} (\beta\left(\frac{x}{\varepsilon}\right) k + a_{ij}\left(\frac{x}{\varepsilon}\right) \alpha_j\left(\frac{x}{\varepsilon}\right) \frac{\partial f}{\partial x_i}) \tilde{\mu} z_k^\varepsilon dx \end{aligned}$$

In (1.22), we have in essence subtracted from the variational formulation of (1.15) appropriate expressions equal to 0 in order to eliminate products of weak convergences.

It is then possible to go to the limit in (1.22) in a way identical to Bensoussan-Lions-Papanicolaou [1], Chapter 1, Section 3. Upon performing this limiting process σ_i and κ_i come out to be:

$$(1.23) \quad \begin{cases} \sigma_i = \overline{(a_{ij} - a_{kj} \frac{\partial \chi_i}{\partial y_k}) \frac{\partial u}{\partial x_i}} - \overline{(a_{ij} \alpha_j - a_{kj} \alpha_j \frac{\partial \chi_i}{\partial y_k})} \tau \\ \kappa_i = \overline{(\lambda_{ij} - \lambda_{kj} \frac{\partial \theta_i}{\partial x_k}) \frac{\partial \tau}{\partial x_i}} \end{cases}$$

Determining v requires some extra effort and the use of Ψ . One defines g^ϵ to be:

$$(1.24) \quad g^\epsilon = 1 + \epsilon \Psi\left(\frac{x}{\epsilon}\right)$$

then it satisfies, for any ω in $H_0^1(\Omega)$:

$$(1.25) \quad \int_{\Omega} a_{ij} \left(\frac{x}{\epsilon}\right) \frac{\partial g^\epsilon}{\partial x_j} \frac{\partial \tilde{\omega}}{\partial x_i} dx = \int_{\Omega} a_{ij} \left(\frac{x}{\epsilon}\right) \alpha_j \left(\frac{x}{\epsilon}\right) \frac{\partial \tilde{\omega}}{\partial x_i} dx$$

Going through the same procedure as in (1.22) but with μ equal to 0 and w_k^ϵ replaced by g^ϵ , we determine v to be:

$$(1.26) \quad v = \overline{(a_{ij} \alpha_j - a_{ij} \frac{\partial \Psi}{\partial y_j}) \frac{\partial u}{\partial x_i}} + \overline{(a_{kj} \alpha_j \frac{\partial \Psi}{\partial y_k})} \tau$$

Defining a_{ij} , A_i , B_i , λ_{ij} , γ_i , σ to be:

$$(1.27) \quad \left\{ \begin{array}{l} a_{ij} = \overline{a_{ij} - a_{kj} \frac{\partial \chi_i}{\partial y_k}} \\ A_i = \overline{a_{ij} \alpha_j - a_{kj} \alpha_j \frac{\partial \chi_i}{\partial y_k}} \\ B_i = \overline{a_{ij} \alpha_j - a_{ij} \frac{\partial \psi}{\partial y_j}} \\ \lambda_{ij} = \overline{\lambda_{ij} - \lambda_{kj} \frac{\partial \theta_i}{\partial y_k}} \\ \gamma_i = \overline{a_{ij} \alpha_j} - A_i \\ \sigma = \overline{a_{kj} \alpha_j \frac{\partial \psi}{\partial y_k}} \end{array} \right. ,$$

it can be shown, using (1.19), that a_{ij} and λ_{ij} are symmetric positive definite, hence invertible, that A_i and B_i are equal and that σ is positive.

We set:

$$(1.28) \quad \alpha_i = a_{ik}^{-1} A_k = a_{ik}^{-1} B_k$$

Recalling (1.18), (1.23), (1.26)-(1.28) yields:

$$(1.29) \quad \left\{ \begin{array}{l} \bar{\rho} \lambda^2 u - a_{ij} \left(\frac{\partial^2 u}{\partial x_i \partial x_j} - \alpha_j \frac{\partial \tau}{\partial x_i} \right) = \bar{\rho} (\lambda f + g) \\ (\bar{\rho} + \sigma) \lambda \tau - \lambda_{ij} \frac{\partial^2 \tau}{\partial x_i \partial x_j} + \lambda a_{ij} \alpha_j \frac{\partial u}{\partial x_i} = \bar{\rho} k + \overline{a_{ij} (y) \alpha_j (y) \frac{\partial f}{\partial x_i}} \end{array} \right. ,$$

and, in view of the properties of the a_{ij} 's and λ_{ij} 's, the Dirichlet form associated to (1.29) is strictly coercive on $(H_0^1(\Omega))^2$, hence (1.29) admits a unique solution in $(H_0^1(\Omega))^2$. Then, using (1.14), we end up with the following proposition:

Proposition 1.2

$R_\lambda(A_\varepsilon)F$ converges weakly in $(H_0^1(\Omega))^3$ to the unique solution in $(H_0^1(\Omega))^3$ of:

$$(1.30) \quad \begin{cases} \lambda u - u_t = f \\ \lambda \bar{\rho} u_t - a_{ij} \left(\frac{\partial^2 u}{\partial x_i \partial x_j} - \alpha_j \frac{\partial \tau}{\partial x_i} \right) = \bar{\rho} g \\ \lambda (\bar{\beta} + \sigma) \tau - \lambda_{ij} \frac{\partial^2 \tau}{\partial x_i \partial x_j} + a_{ij} \alpha_j \frac{\partial u_t}{\partial x_i} = \bar{\beta} k + \gamma_i \frac{\partial f}{\partial x_i} \end{cases}$$

We then define A to be:

$$(1.31) \quad A = \begin{pmatrix} 0 & 1 & 0 \\ \frac{1}{\bar{\rho}} a_{ij} \frac{\partial^2}{\partial x_i \partial x_j} & 0 & -\frac{1}{\bar{\rho}} a_{ij} \alpha_j \frac{\partial}{\partial x_i} \\ 0 & -\frac{1}{\bar{\beta} + \sigma} a_{ij} \alpha_j \frac{\partial}{\partial x_i} & \frac{1}{\bar{\beta} + \sigma} \lambda_{ij} \frac{\partial^2}{\partial x_i \partial x_j} \end{pmatrix}$$

It is simply a matter of reproducing the proof of proposition 1.1, but with constant coefficient this time to show that A generates a semi-group of operators S(t) such that:

$$(1.32) \quad \|S(t)\| \leq \alpha', \text{ for any } t \geq 0$$

Renaming α^{-1} the maximum of α' and α'^{-1} , we deduce from proposition 1.2 and (1.32) the following corollary.

Corollary (1.2)

$R_\lambda(A_\varepsilon)F$ converges weakly in $(H_0^1(\Omega))^3$ to $R_\lambda(A)F$ where:

$$(1.33) \quad F = (f, g, \frac{\bar{\beta}k + \gamma_i \frac{\partial f}{\partial x_i}}{\bar{\beta} + \sigma})$$

Now, (1.8) implies that, for any U , there is a bounded subsequence of $S_\varepsilon(t)U$ that converges weak-* in $L_\infty(\mathbb{R}_+, H)$ to $G(t)$ an element of $L_\infty(\mathbb{R}_+, H)$. This is a direct consequence of the separability of $L_1(\mathbb{R}_+, H)$ and of Banach-Alaoglu's theorem (Rudin [8], Chapter 3). Still identifying a sequence with its subsequences, we get that, for any V in H ,

$$(1.34) \quad \int_0^\infty e^{-\lambda t} (S_\varepsilon(t)U, V)_H dt \xrightarrow{\varepsilon \rightarrow 0} \int_0^\infty e^{-\lambda t} (G(t), V)_H dt$$

where $(,)_H$ is the natural inner product on H .

But the resolvent of the generator of a semi-group applied on a vector U is equal to the Laplace-transform of the semi-group acting on U (Yosida [8], Chapter 9) thus:

$$(1.35) \quad \int_0^\infty e^{-\lambda t} (S_\varepsilon(t)U, V)_H dt = (R_\lambda(A_\varepsilon)U, V)_H$$

which itself converges to:

$$(1.36) \quad (R_\lambda(A) \underline{u}, v)_H = \int_0^\infty e^{-\lambda t} (S(t) \underline{u}, v)_H dt$$

Since v is arbitrary, we finally get, using the uniqueness of Laplace transforms of scalar function that:

$$(1.37) \quad G(t) = S(t) \underline{u} \quad (t \geq 0)$$

We have proved in this section the following theorem:

Theorem

The generalized solution of (1.2) with Dirichlet boundary conditions and initial conditions (f, g, k) in H converges weak-* in $L_\infty(\mathbb{R}_+, H)$ to the generalized solution of :

$$(1.38) \quad \begin{cases} \bar{\rho} \frac{\partial^2 u}{\partial t^2} = a_{ij} \left(\frac{\partial^2 u}{\partial x_i \partial x_j} - \alpha_j \frac{\partial \tau}{\partial x_i} \right) \\ (\bar{\rho} + \sigma) \frac{\partial \tau}{\partial t} = \lambda_{ij} \frac{\partial^2 \tau}{\partial x_i \partial x_j} - a_{ij} \alpha_j \frac{\partial^2 u}{\partial t \partial x_i} \end{cases}$$

with Dirichlet boundary conditions and initial conditions

$$(1.39) \quad (f, g, \frac{\bar{\rho} k + \gamma_i \frac{\partial f}{\partial x_i}}{\bar{\rho} + \sigma})$$

Before concluding this section, let us emphasize once more the rather unusual change in initial temperature in (1.39).

2. FAST OSCILLATIONS OF THE TEMPERATURE FIELD

Since, through a L_∞ weak-* type of convergence a rapidly oscillating function (like $e^{it/\varepsilon}$) goes to 0, it is fairly natural to expect a $\frac{t}{\varepsilon}$ dependence of u^ε and τ^ε . This kind of problem is most easily addressed using asymptotic expansion techniques. We have already mentioned the semi-heuristic character of this section so that we will not dwell on the restrictions to the problem that would make the argument totally rigorous.

Recalling (1.2) we now suppose that u^ε and τ^ε are functions of both t and $\delta = t/\varepsilon$; ∂_t becomes $\partial_t + \frac{1}{\varepsilon} \partial_\delta$. We then Laplace transform (1.2) with respect to both t and δ , the dual variables being respectively ζ and μ . From now on:

$\hat{}$ will denote the t - Laplace - transform

$\tilde{}$ will denote the δ - Laplace - transform

\vee will denote $\hat{}$ or $\tilde{}$

In order to be able to perform these transformations we need to impose initial conditions on both t and δ . We will set:

$$\begin{aligned}
 u^\varepsilon(x; 0, \delta) &= f(x), & u^\varepsilon(x; t, 0) &= p(x, t) \\
 (2.1) \quad \frac{\partial u^\varepsilon}{\partial t}(x; 0, \delta) &= g(x), & \frac{\partial u^\varepsilon}{\partial \delta}(x; t, 0) &= q(x, t) \\
 \tau^\varepsilon(x; 0, \delta) &= k(x), & \tau^\varepsilon(x; t, 0) &= \Theta(x, t),
 \end{aligned}$$

where f, g, k are as before and p, q, θ are unknown. We get:

$$(2.2) \quad \left\{ \begin{aligned} & \rho\left(\frac{x}{\varepsilon}\right) \left\{ (\zeta^2 \check{u}^\varepsilon - \frac{\zeta f}{\mu} - \frac{g}{\mu}) + \frac{2}{\varepsilon} (\zeta \mu \check{u}^\varepsilon - \zeta \hat{p}) + \right. \\ & \frac{1}{\varepsilon^2} (\mu^2 \check{u}^\varepsilon - \mu \hat{p} - \hat{q}) \} = \frac{\partial}{\partial x_i} (a_{ij}\left(\frac{x}{\varepsilon}\right) \left(\frac{\partial \check{u}^\varepsilon}{\partial x_j} - \alpha_j\left(\frac{x}{\varepsilon}\right) \check{\tau}^\varepsilon \right)) \\ & \beta\left(\frac{x}{\varepsilon}\right) \left\{ (\zeta \check{\tau}^\varepsilon - \frac{k}{\mu}) + \frac{1}{\varepsilon} (\mu \check{\tau}^\varepsilon - \hat{\theta}) \right\} = \frac{\partial}{\partial x_i} \left(\lambda_{ij}\left(\frac{x}{\varepsilon}\right) \frac{\partial \check{\tau}^\varepsilon}{\partial x_j} \right) \\ & \quad - a_{ij}\left(\frac{x}{\varepsilon}\right) \alpha_j\left(\frac{x}{\varepsilon}\right) \left\{ \frac{\partial}{\partial x_i} (\zeta \check{u}^\varepsilon - \frac{f}{\mu}) + \frac{1}{\varepsilon} \frac{\partial}{\partial x_i} (\mu \check{u}^\varepsilon - \hat{p}) \right\} \end{aligned} \right.$$

We seek an expansion of u^ε and τ^ε in the form

$$(2.3) \quad \left\{ \begin{aligned} u^\varepsilon &= \sum \varepsilon^i u_i(x, y, t, \delta) \\ \tau^\varepsilon &= \sum \varepsilon^i \tau_i(x, y, t, \delta) \end{aligned} \right. , \text{ where } y = \frac{x}{\varepsilon} .$$

The dependence of the u_i 's and τ_i 's on y is taken to be y -periodic.

This is always what is assumed when performing double scaling in space in problems related to homogenization.

We also need to control the fast time behaviour of u_i and τ_i . Since we would like them to be oscillating in δ , or, at least, to be such that

$$(2.4) \quad \lim_{T \rightarrow +\infty} \frac{1}{T} \int_0^T u_i(x, t, y, \delta) d\delta \quad (\text{respectively } \tau_i)$$

exist and be finite, we are led, through Wiener's Tauberian theorem (Rudin [6], Chapter 9) to suppose that:

(2.5) $\lim_{\mu \rightarrow 0} \mu \check{u}_1$ (respectively $\mu \check{\tau}_1$) exists and is finite,

and we will furthermore assume that this limit is to be taken point-wise in x and weakly in $H^1(Y)$ with regard to the y dependence.

With these considerations in mind we can proceed to replace $\frac{\partial}{\partial x_i}$ by $\frac{\partial}{\partial x_i} + \frac{1}{\varepsilon} \frac{\partial}{\partial y_i}$ and u^ε and τ^ε by their expansions in (2.2).

We obtain two "series" in ascending powers of ε starting at ε^{-2} ; we identify the factors of each of these powers to 0, one after another. As factor of ε^{-2} we get:

$$(2.6) \quad \begin{cases} \rho(y) (\mu^2 \check{u}_0 - \mu \hat{p} - \hat{q}) = \frac{\partial}{\partial y_i} (a_{ij}(y) \frac{\partial \check{u}_0}{\partial y_j}) \\ \frac{\partial}{\partial y_i} (\lambda_{ij}(y) \frac{\partial \check{\tau}_0}{\partial y_j}) - a_{ij}(y) \alpha_j(y) \frac{\partial}{\partial y_i} (\mu \check{u}_0 - \hat{p}) = 0 \end{cases}$$

Since the Dirichlet form associated to the operator

$$(2.7) \quad D = \rho(y) \mu^2 \cdot - \frac{\partial}{\partial y_i} (a_{ij}(y) \frac{\partial \cdot}{\partial y_j})$$

is strictly coercive on the subspace of $H^1(Y)$ consisting of Y -periodic functions, the first equation of (2.6) has a unique solution in $H^1(Y)$; therefore $\frac{\hat{p}}{\mu} + \frac{\hat{q}}{\mu^2}$ is the solution. Hence

$$(2.8) \quad \check{u}_0 = \frac{\hat{p}}{\mu} + \frac{\hat{q}}{\mu^2}$$

But, in view of (2.5), (2.8) implies that $\hat{q} = 0$ thus \check{u}_0 is equal to $\frac{\hat{p}}{\mu}$ and does not depend on y . Inverting (2.8) we get that \check{u}_0 does not

depend on δ either;

$$(2.9) \quad u_0(x, t) = p(x, t)$$

Then from the second equation of (2.6),

$$(2.10) \quad \check{\tau}_0 = \check{\tau}_0(x, \mu),$$

since the only periodic solution of that equation is a constant with respect to y .

As factor of ϵ^{-1} we get, using (2.8), (2.10):

$$(2.11) \quad \begin{cases} \check{D}u_1 = \frac{\partial a_{ij}(y)}{\partial y_i} \frac{\partial \check{u}_0}{\partial x_j} - \frac{\partial}{\partial y_i} (a_{ij}(y) \alpha_j(y)) \check{\tau}_0 \\ \beta(y) (\mu \check{\tau}_0 - \hat{\theta}) = \frac{\partial}{\partial y_i} (\lambda_{ij}(y) \frac{\partial \check{\tau}_1}{\partial y_j}) + \frac{\partial \lambda_{ij}}{\partial y_i}(y) \frac{\partial \check{\tau}_0}{\partial x_j} - \mu a_{ij}(y) \alpha_j(y) \frac{\partial \check{u}_1}{\partial y_i} \end{cases}$$

Defining χ_i^μ and ψ^μ to be the unique periodic solutions in $H^1(Y)$ of

$$(2.12) \quad \begin{cases} \mu^2 \chi_i^\mu - \frac{\partial}{\partial y_k} (a_{kj}(y) \frac{\partial \chi_i^\mu}{\partial y_j}) = - \frac{\partial a_{ki}}{\partial y_k}(y) \\ \mu^2 \psi^\mu - \frac{\partial}{\partial y_k} (a_{kj}(y) \frac{\partial \psi^\mu}{\partial y_j}) = - \frac{\partial}{\partial y_k} (a_{kj}(y) \alpha_j(y)), \end{cases}$$

we obtain from the first equation of (2.11):

$$(2.13) \quad \check{u}_1 = -\chi_j^\mu \frac{\partial \check{u}_0}{\partial x_j} + \psi^\mu \check{\tau}_0$$

Then, integrating the second equation of (2.11) with respect to y and defining γ_i^μ and σ^μ to be the analogues of γ_i and σ for χ_i^μ and ψ^μ as in (1.27),

$$(2.14) \quad \check{\tau}_0 = \frac{1}{\mu} \frac{\overline{\beta\hat{\phi}} + \gamma_i^\mu \frac{\partial \hat{\phi}}{\partial x_i}}{\overline{\beta} + \sigma^\mu} \stackrel{(\text{def.})}{=} \frac{\hat{\eta}^\mu}{\mu}$$

where $\overline{\quad}$ denotes the Y-average $\int_Y dy$

We introduce Λ_k^μ and H^μ to be the unique periodic solutions in $H^1(Y)$, up to a constant, of:

$$(2.15) \quad \begin{cases} -\frac{\partial}{\partial y_i} (\lambda_{ij}(y) \frac{\partial \Lambda_k^\mu}{\partial y_j}) = - (a_{ij}(y) \alpha_j(y) \frac{\partial \chi_k^\mu}{\partial y_i} - \frac{\beta(y)}{\overline{\beta}} \gamma_k^\mu) \\ -\frac{\partial}{\partial y_i} (\lambda_{ij}(y) \frac{\partial H^\mu}{\partial y_j}) = - (a_{ij}(y) \alpha_j(y) \frac{\partial \psi^\mu}{\partial y_i} - \frac{\beta(y)}{\overline{\beta}} \sigma^\mu) \end{cases}$$

The equations (2.15) are well-posed since the Y-average of the right-hand members do vanish by definition of γ_i^μ and σ^μ .

Recalling $\Theta_j(y)$, we obtain for $\check{\tau}_1$:

$$(2.16) \quad \check{\tau}_1 = -\frac{1}{\mu} \Theta_j(y) \frac{\partial \hat{\eta}^\mu}{\partial x_j} - \Lambda_j^\mu \frac{\partial \hat{\phi}}{\partial x_j} + H^\mu \hat{\eta}_\mu + \text{an arbitrary function of } x \text{ only}$$

Finally, as factor of ε^0 , we get:

$$\begin{aligned} & \left[\rho(y) (\zeta^2 \check{u}_0 - \zeta \frac{f}{\mu} - \frac{g}{\mu} + 2\zeta \mu \check{u}_1) + D\check{u}_2 = \frac{\partial}{\partial y_i} (a_{ij}(y) \frac{\partial \check{u}_1}{\partial x_j}) + \right. \\ & \left. + a_{ij}(y) \frac{\partial^2 \check{u}_0}{\partial x_i \partial x_j} + a_{ij}(y) \frac{\partial^2 \check{u}_1}{\partial x_i \partial y_j} - a_{ij}(y) \alpha_j(y) \frac{\partial \check{\tau}_0}{\partial x_i} - \frac{\partial}{\partial y_i} (a_{ij}(y) \alpha_j(y) \check{\tau}_1) \right] \end{aligned}$$

$$(2.17) \left\{ \begin{aligned} \beta(y) (\zeta \check{\tau}_0 - \frac{k}{\mu} + \mu \check{\tau}_1) &= \frac{\partial}{\partial y_i} (\lambda_{ij}(y) \frac{\partial \check{\tau}_2}{\partial y_j}) + \frac{\partial}{\partial y_i} (\lambda_{ij}(y) \frac{\partial \check{\tau}_1}{\partial x_j}) \\ &+ \lambda_{ij}(y) \frac{\partial^2 \check{\tau}_1}{\partial x_i \partial y_j} + \lambda_{ij}(y) \frac{\partial^2 \check{\tau}_0}{\partial x_i \partial x_j} - a_{ij}(y) \alpha_j(y) \frac{\partial}{\partial x_i} (\zeta \check{u}_0 - \frac{f}{\mu}) \\ &- a_{ij}(y) \alpha_j(y) \frac{\partial}{\partial y_i} (\zeta \check{u}_1) - a_{ij}(y) \alpha_j(y) \frac{\partial}{\partial x_i} (\mu \check{u}_1) - a_{ij}(y) \alpha_j(y) \frac{\partial}{\partial y_i} (\mu \check{u}_2) \end{aligned} \right.$$

We integrate both equations of (2.17) with respect to y ; making use of all the previous results of this section, we get:

$$(2.18) \left\{ \begin{aligned} \bar{\rho} (\zeta^2 \hat{p} - \zeta f - g) + \mu^3 \overline{\rho \check{u}_2} &= a_{ij}^\mu \frac{\partial^2 \hat{p}}{\partial x_i \partial x_j} - a_{ij}^\mu \alpha_j^\mu \frac{\partial \hat{\eta}^\mu}{\partial x_i} \\ \bar{\beta} (\zeta \hat{\eta}^\mu - k) + \mu^2 \overline{\beta \check{\tau}_1} &= \lambda_{ij} \frac{\partial \hat{\eta}^\mu}{\partial x_i \partial x_j} - \\ \mu \lambda_{ij}(y) \frac{\partial \Lambda_k^\mu}{\partial y_j} \frac{\partial^2 \hat{p}}{\partial x_i \partial x_k} + \mu \lambda_{ij}(y) \frac{\partial H^\mu}{\partial y_j} \frac{\partial \hat{\eta}^\mu}{\partial x_i} \\ - \zeta a_{ij}^\mu \alpha_j^\mu \frac{\partial \hat{p}}{\partial x_i} + a_{ij}(y) \alpha_j(y) \frac{\partial f}{\partial x_i} - \zeta \mu \hat{\eta}^\mu - \\ \mu^2 a_{ij}(y) \alpha_j(y) \frac{\partial \check{u}_1}{\partial x_i} - \mu^2 a_{ij}(y) \alpha_j(y) \frac{\partial \check{u}_2}{\partial y_i}, \end{aligned} \right.$$

where a_{ij}^μ, α_j^μ are to χ_j^μ and ψ^μ what a_{ij} and α_j are to χ_j and ψ in (1.27).

We now consider the limit of (2.18) as μ goes to 0. The following result holds:

Proposition 2.1

$\chi_k^\mu, \psi^\mu, \mu \Lambda_k^\mu, \mu H^\mu$ go respectively to $\chi_i, \psi, 0$ and 0 strongly in $H^1(Y)/\mathbb{R}$ as μ goes to 0. Hence $a_{ij}^\mu, \alpha_j^\mu, \gamma_i^\mu, \sigma^\mu$ go to $a_{ij}, \alpha_j, \gamma_i, \sigma$.

The proof of this proposition, which involves some basic estimates in $H^1(Y)/\mathbb{R}$ will not be given here; refer to Francfort [4] for the details.

Proposition 2.1 together with (2.5) is exactly what we need to perform the limiting process. Upon doing so, we come up with a set of two equations for \hat{p} and $\hat{\eta}$ which, together with the limit of (2.14), can be interpreted in the time dependent domain. p and θ satisfy:

$$(2.19) \quad \left\{ \begin{array}{l} \theta(x, t) = \frac{(\bar{\beta} + \sigma)\eta(x, t) - \gamma_i \frac{\partial p}{\partial x_i}(x, t)}{\bar{\beta}} \\ \bar{\rho} \frac{\partial^2 p}{\partial t^2} = a_{ij} \frac{\partial^2 p}{\partial x_i \partial x_j} - a_{ij} \alpha_j \frac{\partial \eta}{\partial x_i} \\ (\bar{\beta} + \sigma) \frac{\partial \eta}{\partial t} = \lambda_{ij} \frac{\partial^2 \eta}{\partial x_i \partial x_j} - a_{ij} \alpha_j \frac{\partial^2 p}{\partial t \partial x_i} \\ p(x, 0) = f, \frac{\partial p}{\partial t}(x, 0) = g, \eta(x, 0) = \frac{\bar{\beta}k + \gamma_i \frac{\partial f}{\partial x_i}}{\bar{\beta} + \sigma} \end{array} \right.$$

where $\hat{\eta}$ is the limit of $\hat{\eta}^\mu$ as μ goes to 0.

It is clear that $\eta(x, t)$ can be identified with $\tau(x, t)$, the homogenized temperature field, and $p(x, t)$ with $u(x, t)$, the homogenized displacement field. Replacing θ by its value in (2.14) we also obtain an expression for the leading term of the asymptotic expansion of τ^ϵ , that is τ_0 ; its δ -Laplace transform satisfies:

$$(2.20) \quad \bar{\tau}_0 = \frac{1}{\mu} \frac{\bar{\beta} + \sigma}{\bar{\beta} + \sigma^\mu} \eta + \frac{(\gamma_i^\mu - \gamma_i)}{\bar{\beta} + \sigma^\mu} \frac{\partial p}{\partial x_i}$$

(2.20) is not invertible in general, but the following proposition holds:

Proposition 2.2

σ^μ and γ_i^μ go to zero as μ goes to $+\infty$.

The proof of this last proposition uses the same estimates as the ones that establish proposition 2.1.

Propositions 2.1 and 2.2 enable us to conclude that, as μ goes to 0, $\tilde{\tau}_0$ goes to η , whereas as μ goes to $+\infty$, $\tilde{\tau}_0$ goes to θ . In a time dependent context, these facts translate into statements on the behavior of τ_0 near infinity and near the origin,

$$(2.21) \quad \begin{cases} \lim_{\delta \rightarrow +\infty} \frac{1}{\delta} \int_0^\delta \tau_0(x, t, \delta') d\delta' = \tau(x, t) \\ \lim_{\delta \rightarrow 0} \frac{1}{\delta} \int_0^\delta \tau_0(x, t, \delta') d\delta' = \theta(x, t) \end{cases}$$

The second equation of (2.21) is consistent with our self imposed δ - initial conditions. The first equation shows that the fast oscillations of the leading term τ_0 of the asymptotic expansion of τ^ε are centered about $\tau(x, t)$, the solution of the homogenized problem.

CONCLUSION

Numerical evidence corroborates the results of section 2 and confirms that fast oscillations are indeed the phenomenon leading to this unusual change in initial data [4].

If seeking a more physical explanation, one could examine the entropy associated with the problem:

$$s^\epsilon = \beta\left(\frac{x}{\epsilon}\right)\tau^\epsilon + a_{ij}\left(\frac{x}{\epsilon}\right)\alpha_j\left(\frac{x}{\epsilon}\right)\frac{\partial u^\epsilon}{\partial x_i}$$

It is fairly straightforward , using the results of section 2 and some of the steps performed there, to show that there is no fast dependence in time of the space average of the leading term in the expansion of s^ϵ . That the macroscopic entropy of the body is a slowly varying quantity appears to be a sound idea and does fit our physical intuition. A fast oscillation in the temperature field is the effect that balances the space oscillations of the strains due to the inhomogenities of the coefficients and allows the entropy to evolve slowly at its own pace. In this respect the unusual initial change in temperature is needed to insure that no fast change in entropy is taking place at time zero.

In contrast with other fast oscillation type problems, the "phase" of the oscillations is not arbitrary but perfectly determined. It also appears that a geometrical optics type ansatz in place of (2.1) will fail since, if the solutions of (2.12) are sums of terms of more than one frequency in δ , the fast oscillations need not be periodic in δ .

To conclude this study let us point out that choosing the entropy as the natural variable in place of the temperature introduces space derivatives of the third order and thereby prohibits a rigorous analysis of the type performed in section 1. A perturbation analysis using double scaling is feasible but eventually leads to reintroducing the temperature field as the proper variable.

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